THE LAST WORD

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Have any two raindrops the same number of molecules?

One day I began to wonder if two raindrops ever contained the same number of water molecules-not approximately the same, but exactly the same. How many molecules are we talking about?

If the size of a large raindrop is 5 mm in diameter (1), it's easy to calculate that the number of water molecules in that raindrop is 2.19 X 10^{21} (Equation 1 & 2).

That is greater by far than the grains of sand on Coney Island beach, which has been estimated at 10^{20} (2).

It dawned on me that there was a simple method to find a definite answer. No guesswork is necessary. The method is based on the following analogy: If there are more trees in a forest than there are leaves on any individual tree, then there must be at least two trees with exactly the same number of leaves. Consider, for example, a forest of 10 trees. We can assign numbers to the amount of leaves on individual trees, i.e., 1 for the first tree, 2 for the second tree, etc., until all of the leaves are distributed over the first nine trees with no duplication. However, we have one tree left, and when we assign a number of leaves from 1 to 9 to it, it must match one of the first nine trees.

And so if there are more raindrops than there are water molecules in any individual raindrop, then there must be at least two raindrops with exactly the same number of water molecules. With a raindrop of 5 mm in diameter, we can calculate that in one gallon of water we have 5.78 X 10^4 raindrops. At .0654 cm3 per raindrop and 3790 cm^3 per gal, we have Equation 3.

This means that in order to have more than 2.19 X 10^{21} raindrops (to exceed the number of water molecules in one raindrop), we need at least 3.79 X 10^{16} gal of water (Equation 4).

How long would it take that much rain to fall on the earth? Would it have happened within the past year? By calling the local library, I learned that the earth's annual rainfall is 36 in. (3). Considering the earth as a sphere with a radius of 4000 miles, the earth's surface area is about 5.61 X 10^{15} ft^2 (Equation 4). A 36-in. rainfall over that area would be equivalent to 1.26 X 10^{17} gal of water because 5.61 X 10^{15} ft^2 X 3 ft = 1.68 X 10^{16} ft^3 which at 0.134 ft^3 per gal translates to 1.26 X 10^{17} gal. At 5.78 X 10^4 raindrops per gal, that turns out to be 7.29 X 10^{21} raindrops, comfortably in excess of the 2.19 X 10^{21} raindrops needed to satisfy our condition (more raindrops per year than there are water molecules in an individual raindrop).

Therefore, during the past year there had to be at least two raindrops that had exactly the same number of water molecules. In fact, since 10.9 in. of rainfall would satisfy the minimum condition, during the past 16 weeks at least two raindrops fell with the same number of water molecules. When we divide this volume by the surface area of the earth, we find the annual rainfall that will contain two raindrops with exactly the same number of molecules (Equations 5 and 6).

Equation 1.

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi (.25\text{ cm})^3 = .0654\text{ cm}^3 = .0654\text{ g}$$

Equation 2.

$$\frac{.0654\text{ g}}{\text{Raindrop}} \times \frac{1}{18.0\text{ g/mole}} \times 6.02 \times 10^{23}\ \frac{\text{Molecules}}{\text{mole}} = 2.19 \times 10^{21}\ \frac{\text{Water molecules}}{\text{Raindrop}}$$

Equation 3.

$$\frac{3790\text{ cm}^3\text{ gal}^{-1}}{.0654\text{ cm}^3\text{ Raindrop}^{-1}} = 5.78 \times 10^4\text{ Raindrops per gal}$$

Equation 4.

$$\frac{5.78 \times 10^4\text{ Raindrops}}{\text{Gallon}} = \frac{2.19 \times 10^{21}\text{ Raindrops}}{x},\ x = 3.79 \times 10^{16}\text{ gal}$$

$$4\pi r^2 = 4\pi \left(4000\text{ miles} \times \frac{5280\text{ ft}}{\text{mile}}\right)^2 = 5.61 \times 10^{15}\text{ ft}^2$$

Equation 5.

$$X\text{ gal} = 2.19 \times 10^{21}\text{ Raindrops} \times \frac{1\text{ gal}}{5.78 \times 10^4\text{ Raindrops}} = 3.79 \times 10^{16}\text{ gal}$$

$$X\text{ ft}^3 = 3.79 \times 10^{16}\text{ gal} \times \frac{1\text{ ft}^3}{7.48\text{ gal}} = 5.07 \times 10^{15}\text{ ft}^3$$

Equation 6.

$$\frac{5.07 \times 10^{15}\text{ ft}^3}{5.6 \times 10^{15}\text{ ft}^2} = .905\text{ ft} = 10.9\text{ in.}$$

If we made the same calculations using a smaller raindrop, we'd have even more duplicates. Not only would a larger number of raindrops tall, but each raindrop would have fewer molecules.

In reality the event occurs much more often than our analysis would indicate. Let's assume that only 2.19 X 10^{21} raindrops have fallen (exactly equal to the number of water molecules in a large raindrop). It is highly improbable that the 2.19 X 10^{21} raindrops are composed of all possible number of water molecules evenly distributed in increments of one molecule up to 2.19 X 10^{21} molecules. And any variation from that even distribution would necessarily result in a match (raindrops with the same number of water molecules).

Once the minimum condition is met (10.9 in. of rainfall, equivalent to 2.19 X 10^{21} raindrops), every additional drop is a potential match for some previously fallen raindrop. Actually when the minimum condition is doubled (i.e. 2 x 10.9 in. or 21.8 in. of rainfall), we find that at least three raindrops must have the same number of water molecules. In that situation, if only three raindrops have the same number of water molecules, then all the other raindrops must be matched pairs.

But because we receive more than triple the 10.9 in. (36-in. annual rainfall), we know that at least four raindrops fell with exactly the same number of water molecules during the year. With a maximum of four raindrops, each having exactly the same number of water molecules, we find that all the other raindrops must have been matched triplets (sets of three raindrops with the same number of water molecules).

Thus considering all of the matchings (pairs, triplets, and quartets) possible, my question must be rephrased--Does any raindrop have its own unique number of water molecules?

References
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(3) "Collier's Encyclopedia": MacMillan Educational Co.: New York, 1981; VoL 19, p. 652.

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